EXERCISE 1 (5 marks)

1-The complex plane is related to a direct orthonormal coordinate system. We consider the points A, B -and C of respective affixes: $z_A = 1 + i\sqrt{3}$, $z_B = 1 + i$ and $z_C = 2i\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12} \right)$.

Justify whether each of the statements a), b), c) is true or false.

a. $\arg(z_C) = \dfrac{\pi}{12}$.   0.5mark

b. $\dfrac{z_A}{z_B}$ in algebraic form is $\dfrac{\sqrt{3}+1}{2} + i\dfrac{\sqrt{3}-1}{2}$.   0.5 mark

c. $\dfrac{z_A}{z_B}$ in trigonometric form is $\sqrt{2}\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12} \right)$ 0.5 mark

2. The complex plane is referred to a direct orthonormal coordinate $(O; \vec{u}, \vec{v})$ system.

Consider the points, $A(-i)$, $B(3)$, $C(2+3i)$ and $D(-1+2i)$..

a. Give the coordinate of the midpoints of the segments $[AC]$ and $[BD]$. What can we deduce of the quadrilateral $ABCD$?   1 mark

b. Geometrically interpret the modulus and the argument of $\dfrac{z_C - z_A}{z_D - z_B}$. Calculate

$\dfrac{z_C - z_A}{z_D - z_B}$.   1 mark

c. Deduce from b. the properties verified by the diagonals of $ABCD$. What is the nature of $ABCD$?   1 mark

EXERCISE 2 (5 marks)

The plane P is provided with an orthonormal coordinate system $(O; \vec{i}, \vec{j})$ (graphic unit 3 cm)

1. Consider the function defined on $[0, +\infty[$ by: $\begin{cases} f(x) = \dfrac{\ln(x+1)}{x} & \text{si } x > 0 \\ f(0) = 1 \end{cases}$:

Show that f is continuous at 0.   0.5 mark

2. a. Give the sense of variation of the function g defined by

$g(x) = \ln(1+x) - \left(x - \dfrac{x^2}{2} + \dfrac{x^3}{3} \right)$.   0.5 mark

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Calculate g (0) and deduce that on $\mathbb{R}^+$: $\ln(1+x) \le \left(x - \dfrac{x^2}{2} + \dfrac{x^3}{3} \right)$. 　　　　　0.5 mark

b. By a similar study, show that if, $x \ge 0$, then $\ln(1+x) \ge x - \dfrac{x^2}{2}$.

c. Establish that for all strictly positive x we have $-\dfrac{1}{2} \le \dfrac{\ln(1+x) - x}{x^2} \le -\dfrac{1}{2} + \dfrac{x}{3}$.

Deduce that f is differentiable at zero and that $f'(0) = -\dfrac{1}{2}$ 　　　　　1.5 mark

3. a. Let h be the function defined on $[0, +\infty[$ by $h(x) = \dfrac{x}{x+1} - \ln(1+x)$.

Study its sense of variation and deduce the sign of h on $[0, +\infty[$. 　　　　　0.5 mark

b. Show that on, $[0, +\infty[$, $f'(x) = \dfrac{h(x)}{x^2}$. 　　　　　.　　　　　0.5 mark

c. Draw up the variation table of f by specifying the limit of f in $+\infty$ 　　　　　0.5 mark

d. Denote by C the graphical representation of f. Construct the tangent T to C at the point of abscissa 0. Show that C admits an asymptote. Draw curve C. 　　　　　0.5 mark

EXERCISE 3 (5 marks)

The table below describes the average number y of objects that a worker starting to work on an assembly line produces in one day, on the xth day that he works on that line.

x_i	1	3	5	7	9
y_i	27	41	46	48	49

A. In this part, the functions of the calculator will be used for the statistical calculations (the details of the calculations are not required).

1. The plane P is provided with an orthogonal reference of graphic units 1 cm for a day on the abscissa and 1 cm for 5 objects on the ordinate.

Plot a scattered diagram of the points associated with the statistical series (xi, yi).
　　　　　1 mark

2. Determine the coordinates of the mean point G of this points and place it on the previous graph. 　　　　　1 mark

3. a. Determine the linear correlation coefficient of the statistical series (xi, yi) correct to 10^{-2}. 　　　　　1 mark

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b. Give a regression line (d) of y on x by the least squares method.

Represent the line (d) on the previous graph.   1 mark

c. What day must the worker produce 83 objects?   1 mark

Exercise 4 (5 marks)

Urn A contains four red balls and six black balls. Urn B contains a red ball and nine black balls. The balls are indistinguishable to the touch.

Part A

A player has a six-sided die, perfectly balanced, numbered from 1 to 6. He throws it once: if he rolls 1, he draws a ball at random from urn A, otherwise he draws one at random from urn B.

1. Let R be the event "the player gets a red ball". Show that $p(R) = 0.15$.   0.75mark

2. If the player rolls a red ball, is the probability that it comes from A greater than or equal to the probability that it comes from B?   0.75mark

Part B

The player repeats the test described in part A twice, under identical and independent conditions (that is to say that at the end of the first test, the ballot boxes return to their initial composition).

Let x be a non-zero natural number.

During each of the two tests, the player wins x euros if he obtains a red ball and loses two euros if he obtains a black ball.

We denote by G the random variable corresponding to the algebraic gain of the player in euros at the end of the two tests. The random variable G therefore takes the values $2x$, $x-1$ and - 4.

1. Determine the probability law of G.   1.5mark

2. Express the expectation E (G) of the random variable G as a function of x.   1.5mark

3. For which values of x do we have E (G)> 0?   1mark

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